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| **Characterizing the Performance of an Air Traffic Control Radar Installation** | Grade Level(s): 9-12  Subject(s): Pre-Calculus  Author: Nikhil Joshi  Industry Consultant(s): Mike Pisaruck, Boeing |

# Problem Statement:

*An airliner has gone missing. It was last tracked by air traffic control in the Indian Ocean, but additional information may suggest it traveled further. Family and friends of the victims are anxious to learn more about the incident, while engineers are puzzled by the disappearance of a modern airplane. With mounting public pressure, the airline company is under the gun to locate the missing airplane as quickly as possible.*

When the secondary radar system of an aircraft fails or is disabled, how well can positional data on the aircraft be determined using only primary radar? What aspects of primary radar technology should we focus on to improve the precision of detecting an airplane?

# Conceptual Storyline: Pre-Calculus

A conceptual storyline is the teacher’s description of the ideas, concepts and practices that students will develop in this lesson/unit, how they relate to each other, and how they will build on each other chronologically. It helps the teacher think through the logical process (aka storyline).

Example: Reasoning and justification are central concepts in math where mathematical arguments and decisions are based on knowns to develop insights into unknowns. Proportional reasoning is a concept where measurements are described in relation to each other. Students may still be thinking additively (this dimension is 3 units bigger, so the other dimension would also be 3 units bigger – rather than proportionally bigger). We will pose a Yeti problem, showing the outline of a footprint. We ask students to estimate how big Yeti might be if this is his footprint. The group will brainstorm ideas of the degree to which Yeti is proportional to people (look like a person but bigger and hairier). Then students will investigate relationships of human measures (such as foot size to height, foot size to weight, or other relationships) to anticipate Yeti’s size. Students will justify their solution strategies and decisions and share with each other. The class will determine which approach is most robust and likely, or if a modification based on collective work could create a more robust reason.

Too often important technical concepts are overly simplified when explained to students. Though this may be necessary to quickly explain the big idea, complexity related to real-world implementation usually isn’t even mentioned, so students fail to appreciate the work required to take something from idea to implementation.

The goal of this project is for students to take a simple idea, radar, and go through the steps required to apply the idea to solve a realistic problem by applying pre-calculus to account for the complexity and imperfections of the real world.

This project is a summative assessment of student learning. To fully take advantage of the projects students should have completed studying polynomials, rational functions in general, inverse functions, right triangle and unit circle treatment of trigonometry, and exponentials and logarithms.

Consider the usual explanation of radar, which says that the distance to an object can be determined by transmitting a radio pulse and waiting to receive the radio echo, and dividing the echo-time by twice the speed of light. The application of this idea usually involves having the student solve some basic plug-and-play problems to determine echo time and distance to target.

To more accurately model how the radar concept is applied to detect targets one or more of the following need also be considered.

* Slant range vs. horizontal range
* Curvature of the Earth (height above sea level and downfield range)
* Maximum range due to transmitter power vs. receiver sensitivity
* Signal loss due to inverse square law, atmospheric absorption
* Signal loss due to target radar cross-section
* transmitter/receiver duplexing and maximum and minimum detection ranges
* Maximum unambiguous range due to periodic radar pulsing
* Multi-target resolution limits due to pulse width and bearing precision
* Minimum resolution cell due to limitations of hardware

The students must use their understanding of pre-calculus to model a radar installation and characterize its performance capabilities taking into account several to many of the factors above. Based on their analysis the students should be able to characterize the size of the radar’s resolution cell in terms of and re-express this value into a volume expressed in cubic kilometers as a function of range, i.e. .

Time permitting, based on their mathematical model the students can construct a spreadsheet and analyze their model to determine how sensitive the minimum resolution cell is to changes in system parameters. Ideally they will propose the system property most worth improving to minimize the resolution cell.

# Project Overview

This is a summative assessment project with 1 or possibly two lessons.

**Lesson 1** – Required introduction to radar technology and science background on radiation propagation through space.

**Lesson 2** – Optional lecture on how to develop Assertion-Evidence slide presentations. If you choose not to use this slide model, or choose a different way for students to demonstrate their understanding, you may substitute your lesson plan here, or choose not to present a second lesson if the students are already familiar with your preferred way to communicate their learning.

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| Criterion | Description |
| Timescale | 2 weeks of in-class team work, 1 week of team homework time to develop deliverables as class moves to next unit. |
| Student Deliverables | A technical presentation describing the development and results of the project. Opportunity for students to turn in rough draft for feedback before final draft due. |
| Teaming | Teams of three, power-matched (teams assigned by class rank, so strong students matched with strong students). |
| Differentiation | Strong teams may layer more realism in their solution, weaker teams less. Students can work on layers that best suit their confidence and ability level since many of the layers are relatively independent. Strong teams can be mostly left alone to work independently, leaving more teacher time to work with weaker teams. |
| Teacher Role | Meet with each team twice during in-class phase and at least once during homework phase to answer questions and offer guidance. Teacher will circulate during project work time to monitor progress and assist. |
| Grading | Graded using technical content and presentations rubrics. Counts as a quarter-scale unit grade (e.g. 25 points vs. 100 for unit grade). |

## Learning Standards

### Common Core Standards

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| **Conceptual Category** | **Domain** | **Clusters and Standards** |
| *Number and Quantity* | Quantities | Reason quantitatively and use units to solve problems  *N-Q.A.1, N-Q.A.2, N-Q.A.3* |
| *Algebra* | Seeing Structure in Expressions | Write expressions in equivalent forms to solve problems  *A-SSE.B.3a, A-SSE.B.3b, A-SSE.B.3c, A-SSE.B.4* |
| Arithmetic with Polynomials and Rational Expressions | Rewrite rational expressions  *A-APR.A.1, A-APR.B.2, A-APR.B.3, A-APR.C.4, A-APR.D.6* |
| Creating Equations | Create equations that describe numbers or relationships  *A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4* |
| Reasoning with Equations and Inequalities | Understand solving equations as a process of reasoning and explain the reasoning  Solve equations and inequalities in one variable  *A-REI.B.3, A-REI.B.4a, A-REI.B.4b* |
| *Functions* | Interpreting Functions | Understand the concept of a function and use function notation  Interpret functions that arise in applications in terms of the context  *F-IF.A.1, F-IF.A.2, K, F-IF.A.3, F-IF.B.4, F-IF.B.5, F-IF.B.6* |
| Building Functions | Build a function that models a relationship between two quantities  Build new functions from existing functions  *F-BF.A.1a, F-BF.A.2, F-BF.B.3, F-BF.B.4a* |
| Linear, Quadratic, and Exponential Models | Construct and compare linear, quadratic, and exponential models and solve problems  *F-LE.A.1a, F-LE.A.1b, F-LE.A.1c, F-LE.A.2, F-LE.A.3, F-LE.A.4* |
| Trigonometric Functions | Model periodic phenomena with trigonometric functions  *F-TF.B.5* |
| *Geometry* | Similarity, Right Triangles, and Trigonometry | Define trigonometric ratios and solve problems involving right triangles  *G-SRT.C.6, G-SRT.C.7, G-SRT.C.8* |
| Modeling with Geometry | Apply geometric concepts in modeling situations  *G-MG.A.1, G-MG.A.2,G-MG.A.3* |

What standards (content and practices) are you addressing in this unit/lesson(s)?

Ex. NGSS HS-LS4-4. Construct an explanation based on evidence for how natural selection leads to adaptation of populations.

CCSS Math HS N-RN. Extend the properties of exponents to rational exponents.

### 21st Century Skills

CRITICAL THINKING AND PROBLEM SOLVING

* *Reason Effectively* - Use various types of reasoning (inductive, deductive, etc.) as appropriate to the situation
* *Use Systems Thinking* - Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems
* *Make Judgments and Decisions* - Interpret information and draw conclusions based on the best analysis
* *Solve Problems* - Solve different kinds of non-familiar problems in both conventional and innovative ways

COMMUNICATION AND COLLABORATION

* *Communicate Clearly* - Articulate thoughts and ideas effectively using oral, and written communication skills in a variety of forms and contexts
* *Collaborate with Others* - Assume shared responsibility for collaborative work, and value the individual contributions made by each team member

### NGSS Standards:

What standards (content and practices) are you addressing in this unit/lesson(s)?

Ex. NGSS HS-LS4-4. Construct an explanation based on evidence for how natural selection leads to adaptation of populations.

CCSS Math HS N-RN. Extend the properties of exponents to rational exponents.

CCR Reading Anchor #7: Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.

* RST.6-8.7: “Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).”
* RST.9-10.7: “Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
* RST.11-12.7: “…evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.”

Habits of mind and ways of working together help students build their capacity for workplace expectations. What 21st century skills will students be applying in this lesson or unit (e.g. Communication, Collaboration, Critical Thinking, Creativity)?

## Locally and/or Personally Relevant for Students:

How do students build on their understanding of their school community or on what matters to students? Are there ways to make a strong connection to women or underrepresented minorities in STEM fields – to increase proportionate representation of those groups in STEM?

At Raisbeck Aviation High School, instruction and projects are done primarily in the context of aviation and aerospace. The vast majority of students are interested in aerospace and STEM applications in aerospace (and aerospace applications of STEM). As we are a school of choice, female and minority students have declared their existing interest in aerospace and STEM fields by applying and their intent to pursue STEM degrees and employment in the future.

In more general high school environments connections can be made on a more personal level – may have student have travelled on commercial airplanes or may do so in the future. Understanding how governments and corporations ensure safe aircraft operations is something they may not think about but is clearly relevant to their lives and the lives of friends and family.

## Connections to career and educational pathways:

How will students learn about connections to career and educational pathways into the unit/lessons?

College majors and careers associated with the context will be referred to during the hook and introductory lesson. The context of the problem clearly references career pathways from electrical engineering, applications of physics, and governmental regulations and organizations such as the FAA and Air Traffic Control (ATC). Career and educational pathways such as these are pervasive at Raisbeck Aviation High School.

# Lesson 1: Overview of ATC Radar Technology

This lesson is composed of a brief lecture describing how primary radar works. The concepts will include diagrams and description of

1. Radio frequency energy, speed, diffuse reflection, and backscatter (echo)
2. Radar hardware, including antennas, transmitters, receivers, and duplexing
3. Signal timing, including pulse widths, pulse repetition, duplex switching time
4. Environmental impact on signal strength

## Problem Statement:

restate problem statement and how each lesson fits into problem statement

When the secondary radar system of an aircraft fails or is disabled, how well can positional data on the aircraft be determined using only ATC primary radar? In order to answer this question, the students must understand the construction and operational principles of a modern ATC radar installation.

Students will need to mathematically model the disparate components, expressing relationships with mathematical components and relating abstract concepts and data such as timing signals to physical actions in the environment such as energy transmission and reflection. This lesson is designed to provide the technical background and terminology to get started.

## Learning Objectives:

By the end lesson the students should be able to define and explain technical concepts and terminology related to ATC radar.

## Lesson Standards

This introductory lesson addresses the NGSS standards mentioned above.

## Materials:

1. Segment from MH370 Nova episode to motivate and introduce the project context. (“the hook”)
2. Lecture describing how a primary radar system operates
3. Handouts of Radar Technology presentation
4. ATC Radar data sheet with minimum information required to approach project (e.g. transmitter and receiver metrics, angular dispersion metrics for radar beam, etc.)
5. Problem Selection Form for students to propose the radar problem they have identified and chosen to solve.
6. Problem Solution Sheet for each student to fill out illustrating their solution to their selected problem. If a student chooses to solve more than one problem they must submit separate forms for each solution.

## Lesson Preparation:

Teacher should

1. Review the radar resources below to familiarize themselves with the technology, problems and solutions.
2. Review information on assertion-evidence slide models if they choose to use this to report results (highly recommended)
3. Watch the full NOVA video on MH370 (“Why Planes Vanish”) to select sections they believe provide the best hook for their class. The NOVA video can be found on and streamed from YouTube with some searching.

This is a summative project. Students should have completed studying polynomial and rational functions, right angle trigonometry, unit circle trigonometry, and exponentials and logarithms.

## Time Required:

Seven classroom days and an additional five homework days to develop the team slide deck of solutions. If online collaboration from home is not a viable option, additional class days could certainly be substituted.

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| Class 1 (50 min) | Video Hook and Radar overview lesson (lesson 1) |
| Class 2 (50 min) | Students brainstorm possible problems and each select one problem they wish to solve Students fill out the Problem Selection Form and return to teacher. Alternately, teacher builds a Google Form that students can fill out and submit online. |
| Classes (3-6) (50 min each) | Work days, teacher meets with teams to gauge progress and offer differentiated guidance. |
| Class 7 (50 min) | Introduction to Assertion-Evidence Slide Design (lesson 2) |
| HW (Days 7-11) | Team members collaborate on project slide deck as homework. Google docs used to share presentation contents between students and teacher. |

## Grouping of students for instruction:

Describe how students will be divided into groups, if applicable (random, ability, interest, social purposes, etc.) Will students have roles? If so, how will roles be assigned? How will students learn their roles?

Teams of three, power-matched (teams assigned by class rank, so strong students matched with strong students). Strong teams may layer more realism in their solution, weaker teams less. Students can work on layers that best suit their confidence and ability level since many of the layers are relatively independent. Strong teams tend to work independently and need minimal teacher support, leaving more teacher time to work with weaker teams.

Certainly every teacher knows their students and classes best, so teaming is flexible. Nothing about this projects favors one teaming strategy over another.

# Procedure:

## Understand The Problem:

Describe how you will launch your problem (e.g., worker from your externship will pose the problem in person, skype or via video; the teacher presents the students with a letter from a company)? In this portion of the lesson, students will work towards a common understanding of what the problem is and what they need to know in order to solve the problem.

### Introduction/Problem Launch

An excerpt from the NOVA episode on the MH370 disappearance will be shown. Most students at RAHS should already be familiar with this event.

### Brainstorm What Students Know/Need to Know

This is a summative project. Students have already learned and have been assessed on the major mathematical content – polynomials and rational functions, trigonometry (both circular and right-angle), and exponentials and logarithms. The main learning goal is for students to understand how to apply these concepts in conjunction to solve an abstract and complicated problem.

To guide the students toward this realization a whole-class discussion can take place to brainstorm the kinds of real-world complexity that must be modeled. You can ask students which topics that we’ve studied this year could be applied to a particular “layer” of the problem.

### Define/refine the Problem

Students will not be given a problem statement, each class, through guided discussion will develop their own problem statement aligned with the teacher-defined learning objective – essentially provide a more realistic mathematical model to characterize the radar system.

## Explore the Problem:

How will students’ explore multiple ideas, pathways, and challenge their current conceptions? How will all students access the information/context? The students (groups) will develop multiple solutions to the problem based on their evidence that will be shared in the next section.

Initial approaches to the problem will be discussed at the class level, then at the team level. Teacher will circulate and answer (or not answer) questions as appropriate. Students have had ample practice throughout the year with the structured problem solving approach (based on Polya’s work) and will apply these techniques to deconstruct the scenario and assign and solve sub-problems.

### Gather and Generate Information

The bulk of the work will be modelling the radar system mathematically and accounting for real-world complexity. The teams will need to research physical constants such as radius of the earth, speed of light, systemic signal loss estimates such as energy absorption by the atmosphere or electronic inefficiencies, and define their own terms such as altitude vs. height, etc.

Different teams will likely layer different amounts of realism into their radar model resulting in different solution fidelity.

### Share Information

Teams will naturally share information in an ad-hoc fashion during the research and development stage of the project. However, midway through the in-class timeline teams can be paired up to share status with each other.

### Generate Possible Solutions

Based on their mathematical radar model teams will determine their theoretical maximum range and ideally their minimum resolution cell. Although aspects of this problem can be researched from sources on the internet, students are responsible for demonstrating the derivation of all pertinent equations used on their solution and defending their work.

## Resolve the Problem:

Students should be able to provide an argument for each of the possible solutions and be given an opportunity to share and critique arguments. How will students reflect upon and share what they’ve learned? How will students synthesize their learning? If there are presentations involved with this PBL, how do you plan to help the non-presenters learn from presentations?

### Determine Best Fit solution

Students will reflect on their model and propose and defend at least one modification to the radar system which they think is the most important in improving the accuracy and detection. For example they may propose improving transmitter power to increase maximum detection range. They must mathematically demonstrate how much improvement is gained for each percentage change in their modified criterion (e.g. each 10% increase in transmitter power will result in 3% improved maximum range).

It is expected that there will be a diverse set of solutions presented by various teams based on this requirement.

### Present the Solution

Students will present their solution in the form of an assertion-evidence slide deck.

### Debrief the problem

As this will be the first year the project will be attempted, the students will be solicited for feedback on what aspects went well, and which areas need to be improved and why.

### Assessment:

How will you assess student learning during the problem? Will there be a final product? Will the final product criteria be clear for students from the beginning? Will there be both whole group elements and individual accountability? Attach appropriate rubrics

Formative assessment will be made as the teacher circulates during work time. Each student must submit a solution sheet for their selected problem. This solution will be assessed using the technical content rubric and can be used as an individual grade element for the project.

Each time will deliver a technical slide deck that complies with the assertion-evidence slide model. The slide deck will be assessed using the technical presentation rubric. Students will specify which slides they were responsible for within the deck using the slide notes section. Each student will be assessed on their slides and the team will receive a team grade for the slide deck as a whole.

### Accommodations:

Describe special accommodations for any students with significant exceptional needs (i.e. visual impairment, deafness, physical impairments, etc.)

Typical accommodations may involve extra time, more specific written directions, and allowed to use laptop to complete work etc. More extreme exceptional needs can be addressed with alternate deliverables (oral report, hand drawn slides, interview, etc.).

### Extensions:

Describe possible ways to extend the lessons, if time allows.

Each student must provide at least one layer of realism to the project. If this is accomplished early or easily, there are many other levels of complexity that can be added, such as factoring in range resolution, target resolution, etc. Students can add another layer at the teacher’s discretion, for example, after the teacher approves the student’s previous solution.

# References/Resources:

Attach any materials students will use during the lesson; e.g., handouts, questions to answer, and worksheets.Acknowledge your sources. Give credit to the person who created the idea for the instructional plan, including yourself.  You might use language such as "Instructional Plan adapted from \_\_\_\_\_”; “Instructional Plan Consultants (not responsible for the content of this instructional plan): \_\_\_\_\_\_\_”; and/or “Instructional Plan Created by \_\_\_\_\_” Cite scripted materials/curriculum if appropriate.

Instructional Plan adapted by N. Joshi and based on “Radar Basics” by Wolff (see beow).

Special thanks to Mike Pisaruk from Boeing for his insight on radar technology and help on the instructional plan.

All presentations in my class must comply with the Assertion-Evidence Slide Design Model. I’ve had excellent results using this for my classes, so much so that I’ve discovered my students teach other kids how to build assertion-evidence slide decks.

See <http://www.craftofscientificpresentations.com/assertion-evidence-approach.html> for more information.

**Resources:**

1. Wolff, C. (1998, November). Radar Basics. Retrieved Feb 11, 2016, from <http://www.radartutorial.eu/index.en.html>
2. Jenn, D. Radar Fundamentals. Retrieved Feb 11, 2016, from http://faculty.nps.edu/jenn/
3. Smarter Balanced Grade 11 Mathematics Specifications Document Overview. (2014, December). Retrieved August 6, 2015, from <http://www.k12.wa.us/Mathematics/pubdocs/Grade11ClaimDistribution.pdf>
4. P21 Framework Definitions. (2015, May). Retrieved April 30, 2016, from http://www.p21.org/storage/documents/docs/P21\_Framework\_Definitions\_New\_Logo\_2015.pdf
5. Connections to the Common Core State Standards for Literacy in Science

and Technical Subjects. Retrieved May 5, 2016, from <http://www.nextgenscience.org/get-to-know>

1. Rethinking Presentation Slides: The Assertion-Evidence Structure, <http://www.craftofscientificpresentations.com/assertion-evidence-approach.html>

WABS STEM PBL Unit/Lesson Plan Template

Description:

Problem-based learning (PBL) is focused, experimental learning organized around the investigation and resolution of messy and real world problems. The Final Project will allow you to organize your lesson in a problem solving environment where students engage in learning in relevant and connected ways. Teachers function as a coach to guide student inquiry and facilitate learning to deeper levels of understanding for your students.

Research indicates that PBL is a superior pedagogy for promoting student engagement in the learning process. Torp and Sage (2002)1 broaden the impact of this pedagogy and confirm that it increases motivation, makes learning relevant to the real-world, promotes higher order thinking and self-regulated learning in students.

Generally, the teacher will present the problematic situation. The problem is ill-structured and messy (multiple sub-problems), not easily solved and does not result in one right answer.  Students engage in active problem solving, and teachers guide and coach. A collaborative environment provides for the sharing of information within and between groups as they work to resolve - some may test and re-resolve - their problems. Authentic assessment compliments the problem solving process.

1 Torp, L., & Sage, S. (2002) Problems as Possibilities:  Problem Based Learning for k16 Education (2nd ed.).  Alexandria, VA:  Association for Supervision and Curriculum Develo